MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

BNA3014 – NUMERICAL ANALYSIS

(All sections / Groups)

12 OCTOBER 2019
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **4 pages** including formulae sheets with **4 questions** only.

2. Attempt **ALL** questions and write your answers in the Answer Booklet provided.

3. Students are allowed to use non-programmable scientific calculators that are permitted to be used in the examination.

Question 1 (25 Marks)

(a) Find the third degree Taylor polynomial approximation for the following function at a=1,

$$f(x)=\frac{1}{2+x}$$ [8 marks]

(b) Consider the following function,

$$f(x)=\sqrt{1+x}-\sqrt{1-x}$$

i. Suggest a function $g(x)$ that will avoid loss of significance error of $f(x)$. [4 marks]

ii. Compute $f(0.25)$ and $g(0.25)$ by using three-significant digits and rounding. [6 marks]

iii. Given that the actual value is 0.252008585, compare the relative error for both calculations in part (ii). [7 marks]

Question 2 (25 Marks)

(a) Using the Newton's method, find the root of the equation $2x^2+5=e^x$ with starting point p_0=3.5. Stop the iteration when $|p_n-p_{n-1}|<1\times10^{-4}$.

Note: Write your answer correct to four-decimal places. [12 marks]

(b) Use Gaussian Elimination without partial pivoting to solve the following system of linear equations.

$$\begin{aligned}4x_1+2x_2-1x_3&=-5\\ \frac{1}{9}x_1+\frac{1}{9}x_2-\frac{1}{3}x_3&=-1\\ x_1+4x_2+2x_3&=9\end{aligned}$$

[13 marks]

Continued .....

Question 3 (25 Marks)

(a) For the following given table:

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	9.9833	4.9667	3.2836	2.4339	1.9177

i. Construct a Newton's divided difference table. [7 marks]

ii. Using table in part (i), write the Newton polynomial $P_4(x)$ and approximate it for x=0.35. [5 marks]

(b) The following table shows approximate after-tax profits, in millions of Ringgit Malaysia (RM) of Axiata Group Berhad for every two years from year 2007 to 2017:

Year (x)	0	2	4	6	8
Profit (y)	360	400	500	580	600

i. Obtain a model that represents this data by using the method of least square. Fit the model equation of the form $y = ae^{bx}$. [11 marks]

ii. Predict the projected profit in 2019 using the model in part (i). [2 marks]

Question 4 (25 Marks)

(a) For the function, $f(x) = \int_1^4 \sqrt{1+x^2}\,dx$

Find the approximation of the integral $f(x)$ by using Simpson's rule taking six equal subintervals. [10 marks]

(b) Solve the following differential equation using Euler's method

$$\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2, \quad y(1) = 0$$

for the range $1 \leq x \leq 3$ using a step size of 0.4.

Note: Use 4 decimal places for the working. [15 marks]

End of Page

A. ERROR ANALYSIS

Taylor Polynomial :

$$P_n(x) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(a)(x-a)^k$$

Maclaurin Series :

(i) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ (ii) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ (iii) $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

(iv) $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ (v) $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n!}$

B. SOLUTION OF NONLINEAR EQUATIONS

Newton-Raphson Method: $p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$

Secant Method: $p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$

C. SOLUTION OF LINEAR SYSTEMS

Gauss-Jacobi Method : $x_i^k = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k-1} - \sum_{j=i+1}^{n} a_{ij} x_j^{k-1} \right]$

Gauss-Seidel Method: $x_i^k = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k-1} - \sum_{j=i+1}^{n} a_{ij} x_j^{k-1} \right]$;

where $x_j^{k-1} = x_j^k$ based on the latest updates

D. INTERPOLATION AND POLYNOMIAL APPROXIMATIONS

Lagrange Polynomial:

$$P_n(x) = \sum_{i=0}^{n} \frac{1}{k!} f(x_i) L_i(x)$$

E. NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

Least square fitting:

(i) Linear : $y = a + bx \quad ; \sum y = na + b\sum x \quad ; \sum xy = a\Sigma x + b\sum x^2$

(ii) Exponential: $y = ae^{bx} \quad ; \sum z = nA + b\sum x \quad ; \sum xz = A\sum x + b\sum x^2$

F. SOLUTION OF DIFFERENTIAL EQUATIONS

Euler Method:

$$y_{n+1} = y_n + hf(x_n, y_n) \quad \text{where } n = 0,1,2,3,\cdots$$

Runge-Kutta second order Method:

$$k_1 = f(x_n, \quad y_n)$$

$$k_2 = f(x_n + h, \quad y_n + hk_1)$$

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$